OPEN MIDDLE STARTER PACK

Thank you for downloading the Open Middle starter pack. We hope it will make you excited to explore how Open Middle problems can help your students learn.

OUR FAVORITE PROBLEMS

The first part of this PDF includes our favorite Open Middle problems from all grade levels from kindergarten through Calculus. We include:

- why we love the problem
- hints you can ask your students when they get stuck
- answer(s) to the problem
- a link to the problem on Open Middle
- the name of the person who created the problem

OPEN MIDDLE WORKSHEET

The second part has the Open Middle Worksheet, which is a tool to help students persevere while problem solving. You can read all about it here but it incentives learning from mistakes and trying again rather than giving up if you make a mistake.

We include:

- English, Spanish, and French versions
- Student and document camera versions

We hope you fall in love with Open Middle problems like we have. If you're on Twitter, be sure to follow us at @openmiddle and check out the hashtag #whyopenmiddle to see what other teachers are saying about our problems.

Sincerely,
Your friends at Open Middle

KINDERGARTEN

Problem

Using the digits 1 to 9 at most one time each, place a digit in each box to make a true statement.

$$\square + \square = \square - \square$$

Why We Love It

This problem gives students opportunities to play with early understandings of equality. Some students may possibly come up with “true” statements such as 3 + 4 = 7 – 1, providing opportunities for addressing misconceptions.

Hints

- How can you represent your equation using pictures?
- What does the equals sign mean?

Answers

There are many answers including:

- 3 + 5 = 9 – 1
- 1 + 2 = 6 – 3
- 4 + 1 = 7 – 2

Problem Source

https://www.openmiddle.com/adding-and-subtracting-within-10/

Problem Creator

Owen Kaplinsky

FIRST GRADE

Problem

Using the digits 1 to 9 at most one time each, place a digit in each box to create a true statement.

□ = □ + □ = □ + □ + □

Why We Love It

We love it because students can model the multiple solutions with Unifix cubes and because students simultaneously know the goal but love the fun challenge.

Hints

- Are there numbers that are helpful to use first or save for last?
- What would a drawing/model look like to match this equation?

Answers

There are 48 answers including:

- 8 = 2+6 = 1+3+4
- 9 = 1+8 = 4+3+2
- 9 = 7+2 = 3+5+1

Problem Source

https://www.openmiddle.com/equivalent-statements/

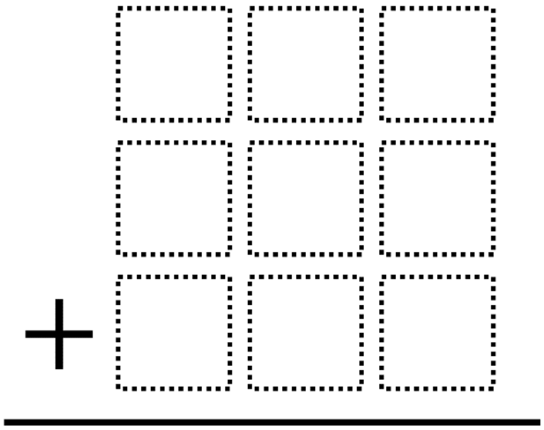
Problem Creator

Molly Rawding

SECOND GRADE

Problem

Using the digits 1 to 9 exactly one time each, place a digit in each box to make the sum as close to 1000 as possible.



Why We Love It

No matter what digits students initially use, you're in the game. The question now becomes, can I get any closer and how? Soon kids can't stop trying to improve upon their previous work.

Hints

- How do the digits you pick for the hundreds place affect your sum?
- How do the digits you pick for the tens and ones place also affect your sum?
- How do you know you can't get any closer to 1000?

Answers

The closest you can get is 999 from answers including:

- 247 + 563 + 189
- 168 + 379 + 452
- 496 + 128 + 375

Problem Source

https://www.openmiddle.com/close-to-1000/

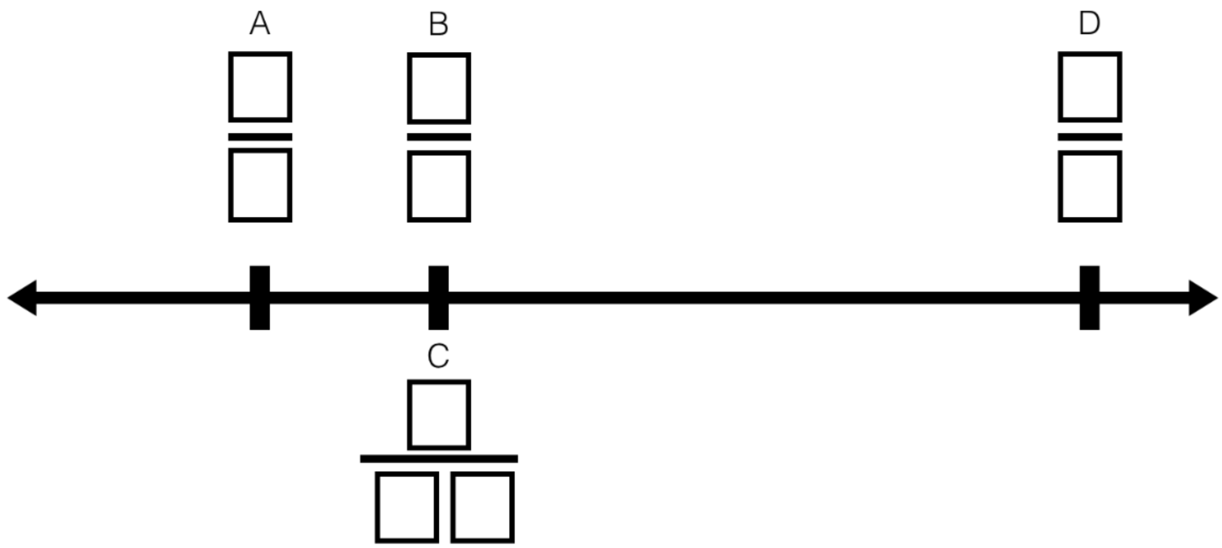
Problem Creators

John Ulbright and Robert Kaplinsky

THIRD GRADE

Problem

Using the digits 1 to 9 exactly once, put a digit in each box to create and place 4 fractions on the number line in the correct order. Fractions B & C are equal.



Why We Love It

We love it because students have so much fun figuring out potential solutions. They ask so many good questions to each other, make mistakes, and spend a long time thinking about numbers on the number line.

Hints

- How do you know if two fractions are equivalent?
- How do you know if fractions are less than or greater than another fraction?

Answers

There are many possible answers including:

- A = 1/7, B = 3/8, C = 9/24, D = 5/6
- A = 1/5, B = 3/9, C = 8/24, D = 6/7
- A = 5/8, B = 3/4, C = 9/12 and D = 6/7. (or D could be 7/6)

Problem Source

Problem Creators

Graham Fletcher, Bowen Kerins, and Kate Nowak

FOURTH GRADE

Problem

Using the digits 0 to 9 at most one time each, place a digit in each box so that each expression is simplified to a different odd number.

$$\square \div (\square - \square)$$

$$\square + \square \times \square$$

$$\square - \square \div \square \times \square$$

Why We Love It

The conversations students have as they develop conceptual understanding are phenomenal! So much great learning happens around discussing misconceptions that come out.

Hints

- How do we know which digits are easier to place?
- How can we tell which operations might affect getting a whole number result for our expression?

Answer

There are many solutions to this problem including:
5 ÷ (8 – 7) = 5
9 + 0 x 6 = 9
3 – 4 / 2 ÷ 1 = 1

Problem Source

https://www.openmiddle.com/order-of-operations-5/

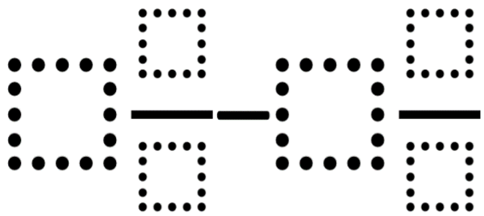
Problem Creator

Molly Rawding

FIFTH GRADE

Problem

Using the digits 1 to 9 at most one time each, place a digit in each box to make the least possible difference.



Why We Love It

This problem is a challenge!!! No matter what digits students first pick, they've got an answer. Now the challenge begins when students try to figure out which digits to alter to make the difference even smaller. Hearing kids argue about whose difference is less than the other is icing on the cake.

Hints

- What does the least possible difference mean? (We're considering this to mean that the two mixed numbers are as close together as possible on a number line)
- How do you know you can't get a lesser difference?

Answer

Assuming that mixed numbers containing improper fractions are allowed, the least possible difference is actually zero when the two mixed numbers are equivalent. Here's one example:

$7\frac{2}{8} - 6\frac{5}{4}$

Problem Source

https://www.openmiddle.com/subtracting-mixed-numbers/

Problem Creator

Robert Kaplinsky

SIXTH GRADE

Problem
Using the digits 1 to 9 at most one time each, place a digit in each box to make a true equation where x has the greatest possible value.

$$\square\square + x = \square\square$$

$$x = \square\square$$

Why We Love It

Hints

- How do the constants' values affect the variable's value?
- Which constant should have a greater value?
- Which constant should have a lesser value?

Answer
The greatest possible value of x is 85 when 12 + x = 97

Problem Source
https://www.openmiddle.com/solving-one-step-equations-2/

Problem Creator
Robert Kaplinsky

SEVENTH GRADE

Problem

Using the digits 1 to 9 at most one time each, place a digit in each box to find the greatest (or least) possible values for x.

$$\square x + \square = \square$$

Why We Love It

Too often when students work on two-step equations, it's hard to tell if they're guessing solutions and plugging them in or whether they deeply understand how to solve an equation. This problem makes guessing and checking very inefficient and forces students to think about how the coefficient and constants affects a two-step equation's solution.

Hints

- How does each constant's value affect the solution's value?
- How does the coefficient's value affect the solution's value?

Answers

Assuming x can be a negative value, 1x + 9 = 2 gives the least possible value of -7. The greatest possible value would be, 1x + 2 = 9

Problem Source

https://www.openmiddle.com/two-step-equations/

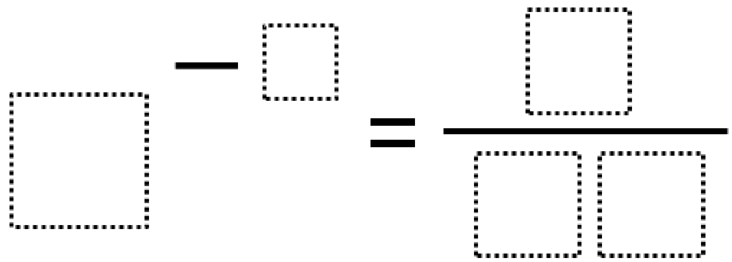
Problem Creators

Audrey Mendivil, Daniel Luevanos, and Robert Kaplinsky

EIGHTH GRADE

Problem

Use the digits 0 to 9 at most one time each, place a digit in each box to make a true statement.



Why We Love It

This problem is a really accessible way to explore understandings and misconceptions around negative exponents. Students can greatly narrow down the possibilities by using their conceptual understanding to realize what bases and exponents would not result in a fraction with a two-digit denominator.

Hints

- How does a negative exponent result in a fractional solution?
- How do we know where the 1 should go?

Answers

There are four possible answers:

- $3^{-2} = \frac{1}{09}$
- $7^{-2} = \frac{1}{49}$
- $8^{-2} = \frac{1}{64}$
- $2^{-3} = \frac{1}{08}$

Problem Source

https://www.openmiddle.com/negative-exponents/

Problem Creator

[Daniel Luevanos]()

ALGEBRA 1

Problem

Using the digits 1 to 9 at most one time each, place a digit in each box to create an equation such that the solution is as close to zero as possible.

$$\square x + \square = \square x + \square$$

Why We Love It

Placing any four digits will create an equation with a solution. Then the challenge begins and students have to develop and use their conceptual understanding to think about how the constants' and coefficients' values affect the solution's value.

Hints

- Which kinds of numbers are closer to zero?
- How does the digit you choose for the constant or coefficient affect the solution's value?

Answers

The closest solution to zero is either 1/8 or -1/8. There are multiple equations with those solutions including:

- 9x + 3 = 1x + 4
- 1x + 7 = 9x + 8

Problem Source

https://www.openmiddle.com/create-an-equation-with-a-solution-closest-to-zero/

Problem Creator

Daniel Luevanos

GEOMETRY

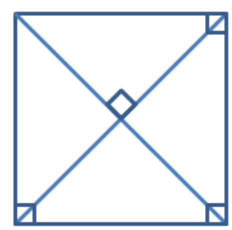
Problem
What is the least number of geometric markings needed to demonstrate that a quadrilateral is a square?

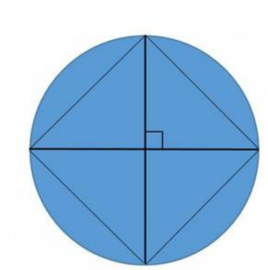
Why We Love It
The debates students will have with one another are why we love our jobs. For example, a student might realize that eight markings (four right angle markings and four equivalent side lengths) would work, but what about seven marking? Six? Five? Less than five? Then the whole room explodes when someone shows how they can do it in one!

Hints

- What makes a quadrilateral a square?
- What markings would make a quadrilateral a square?
- What are the properties of a square that make it unique to the other quadrilaterals?

Answers
Four markings are needed because the three right angles at the vertices prove that the quadrilateral is a rectangle and the right angle at the intersection of the diagonals prove that it is a rhombus. The quadrilateral is a rectangle and a rhombus, so then the quadrilateral must be a square.





However, Amy Gross explained that she started with a circle and had perpendicular diameters. The adjacent endpoints of the diameter are connected with segments to form a quadrilateral, which can be proved to be a square. The issue comes down to how many geometric markings you count.

Problem Source
https://www.openmiddle.com/is-the-quadrilateral-a-square/

Problem Creator
A collaborative effort of Jose De La Torre and Nanette Johnson answer by Ricardo Navarro with help from Robert Kaplinsky

ALGEBRA 2

Problem
Use the integers -9 to 9 at most one time each, place an integer in each box to make a positive real number product and then repeat the process to make a negative real number product. You may use all the integers each time.

$$(\square + \square i)(\square + \square i)$$

Why We Love It
This problem gets so many interesting misconceptions to come out. Initially students may think the problem is impossible or may think that they only have to pay attention to the real number portion. It forces students to build conceptual understand of the multiplication process.

Hints
- What has to happen to the imaginary terms to make a complex number a real number?
- How can we accomplish that?

Answers
There are many answers but what has to happen with all of them is that the products of the real and imaginary terms have to eliminate each other.

An example of a negative product is (-6 + 4i)(3 + 2i). This has a product of -18 + -12i + 12i + 8i^2 which is equivalent to -18 + 8i^2 or -26.

An example of a positive product is (6 + 4i)(3 + -2i). This has a product of 18 + -12i + 12i + -8i^2 which is equivalent to 18 + -8i^2 or 26.

Problem Source
https://www.openmiddle.com/complex-number-products/

Problem Creator
Robert Kaplinsky

PRE-CALCULUS

Problem

Using the digits 1 to 9 at most one time each, place a digit in each box to find the function's greatest possible value.

$$\sin\frac{\square\,\pi}{\square} = \frac{\sqrt{\square}}{\square}$$

Why We Love It

In using this problem with others, it's amazing how many misconceptions come out as well as values that appear to be the greatest possible value… until someone else finds one that's even greater. It leads to great conversations about what's possible and conceptual understanding is developed along the way.

Hints

- What values of pi are more likely to result in a larger value?
- How can equivalent fractions allow us to avoid using a number more than once?

Answers

The greatest possible value is 1 and can be found several equivalent answers including:

- $\sin\frac{3\pi}{6} = \frac{\sqrt{4}}{2}$
- $\sin\frac{3\pi}{6} = \frac{\sqrt{2}}{1}$

Problem Source

https://www.openmiddle.com/sine-functions-2/

Problem Creator

Robert Kaplinsky

CALCULUS

Problem

Using the digits 1 to 9 at most one time each, place a digit in each box to make a solution that is as close to 100 as possible.

$$\int_{\square}^{\square} x^{\square}\, dx$$

Why We Love It

Students may know how to solve a definite integral without having deep understanding of how the bounds and exponents work. This problem forces students to develop and use their conceptual understanding to efficiently find closer and closer solutions.

Hints

- How do the upper and lower bounds determine whether the solution is negative or positive?
- How does the exponent affect the solution's value?

Answers

The solution whose value is 98.666 and is as close to 100 as possible comes from

$$\int_{6}^{8} x^2\, dx$$

Problem Source

https://www.openmiddle.com/definite-integral-3/

Problem Creator

Robert Kaplinsky

Name: ____ Period: ________ Date: _______________

First attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Second attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Third attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Fourth attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Fifth attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Sixth attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

First attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Second attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Third attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Fourth attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Fifth attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Sixth attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Nom: ____ Période: ________ Date: _______________

Première tentative: Points: ____/2 tentative ____/2 explication

Qu'avez-vous appris de cette tentative? Comment votre stratégie changera-t-elle lors de votre prochaine tentative?

Deuxième tentative: Points: ____/2 tentative ____/2 explication

Qu'avez-vous appris de cette tentative? Comment votre stratégie changera-t-elle lors de votre prochaine tentative?

Troisième tentative: Points: ____/2 tentative ____/2 explication

Qu'avez-vous appris de cette tentative? Comment votre stratégie changera-t-elle lors de votre prochaine tentative?

Quatrième tentative: Points: ____/2 tentative ____/2 explication

Qu'avez-vous appris de cette tentative? Comment votre stratégie changera-t-elle lors de votre prochaine tentative?

Cinquième tentative: Points: ____/2 tentative ____/2 explication

Qu'avez-vous appris de cette tentative? Comment votre stratégie changera-t-elle lors de votre prochaine tentative?

Sixième tentative: Points: ____/2 tentative ____/2 explication

Qu'avez-vous appris de cette tentative? Comment votre stratégie changera-t-elle lors de votre prochaine tentative?

Première tentative: Points: ____/2 tentative ____/2 explication

Qu'avez-vous appris de cette tentative? Comment votre stratégie changera-t-elle lors de votre prochaine tentative?

Deuxième tentative: Points: ____/2 tentative ____/2 explication

Qu'avez-vous appris de cette tentative? Comment votre stratégie changera-t-elle lors de votre prochaine tentative?

Troisième tentative: Points: ____/2 tentative ____/2 explication

Qu'avez-vous appris de cette tentative? Comment votre stratégie changera-t-elle lors de votre prochaine tentative?

Quatrième tentative: Points: ____/2 tentative ____/2 explication

Qu'avez-vous appris de cette tentative? Comment votre stratégie changera-t-elle lors de votre prochaine tentative?

Cinquième tentative: Points: ____/2 tentative ____/2 explication

Qu'avez-vous appris de cette tentative? Comment votre stratégie changera-t-elle lors de votre prochaine tentative?

Sixième tentative: Points: ____/2 tentative ____/2 explication

Qu'avez-vous appris de cette tentative? Comment votre stratégie changera-t-elle lors de votre prochaine tentative?

Nombre: ____ Periodo: ________ Fecha: _________________

Primer intento: Puntos: _____/2 intento _____/2 explicación

¿Qué aprendiste de este intento? ¿Cómo cambiará su estrategia en su próximo intento?

Segundo intento: Puntos: _____/2 intento _____/2 explicación

¿Qué aprendiste de este intento? ¿Cómo cambiará su estrategia en su próximo intento?

Tercer intento: Puntos: _____/2 intento _____/2 explicación

¿Qué aprendiste de este intento? ¿Cómo cambiará su estrategia en su próximo intento?

Cuarto intento: Puntos: ____/2 intento ____/2 explicación

¿Qué aprendiste de este intento? ¿Cómo cambiará su estrategia en su próximo intento?

Quinto intento: Puntos: ____/2 intento ____/2 explicación

¿Qué aprendiste de este intento? ¿Cómo cambiará su estrategia en su próximo intento?

Sexto intento: Puntos: ____/2 intento ____/2 explicación

¿Qué aprendiste de este intento? ¿Cómo cambiará su estrategia en su próximo intento?

Primer intento: Puntos: ____/2 intento ____/2 explicación

¿Qué aprendiste de este intento? ¿Cómo cambiará su estrategia en su próximo intento?

Segundo intento: Puntos: ____/2 intento ____/2 explicación

¿Qué aprendiste de este intento? ¿Cómo cambiará su estrategia en su próximo intento?

Tercer intento: Puntos: ____/2 intento ____/2 explicación

¿Qué aprendiste de este intento? ¿Cómo cambiará su estrategia en su próximo intento?

Cuarto intento: Puntos: ____/2 intento ____/2 explicación

¿Qué aprendiste de este intento? ¿Cómo cambiará su estrategia en su próximo intento?

Quinto intento: Puntos: ____/2 intento ____/2 explicación

¿Qué aprendiste de este intento? ¿Cómo cambiará su estrategia en su próximo intento?

Sexto intento: Puntos: ____/2 intento ____/2 explicación

¿Qué aprendiste de este intento? ¿Cómo cambiará su estrategia en su próximo intento?